Thevenin’s Theorem

Thevenin’s Theorem can be used for two purposes:

1. To calculate the current through (or voltage across) a component in any circuit,
   
or

2. To develop a constant voltage equivalent circuit which may be used to simplify the analysis of a complex circuit.

Any linear one-port network can be “replaced with” a single voltage source in series with a single resistor (see Fig. 1 below). The voltage source is called the Thevenin equivalent voltage, and the resistor is called the Thevenin equivalent resistance. This means that the single voltage source and series resistor must behave identically to the actual network it is replacing.

You can use Thevenin’s theorem to solve a complex DC circuit.

![Figure 1](image)

Figure 1  A network replaced with its Thevenin equivalent circuit
The steps used for implementing Thevenin’s Theorem are listed below:

**Step 1**
Remove the resistor, $R$ through which you wish to calculate the current or across which you want to know the voltage. Label these terminals (where the resistor was removed) “a” and “b”. Calculate the voltage that appears across these open terminals. This is called the open circuit voltage or the Thevenin equivalent voltage, $V_{TH}$.

![Figure 2](image)

Let’s consider the example shown in Fig. 2. Use the voltage source, $V_1$, and the voltage dividing network made up of $R_4$, $R_3$, and $R_2$. Here resistor $R_2$ does not influence the voltage that appears across the $a$ and $b$ terminals. This is because no current is drawn through $R_2$ when measuring the voltage across the $a$ and $b$ terminals. This leaves only $R_3$ and $R_4$. What is left looks remarkably like a series circuit. From Kirchoff’s laws we know that the series circuit will divide $V_1$ as given in Eq. 1.

$$V_{TH} = V_1 \frac{R_3}{R_3 + R_4}$$  \hspace{1cm} (1)

**Step 2**
From the open terminals, (“a” and “b”) calculate the resistance “looking back” from the open terminals into the network. Each voltage source must be replaced by a resistor equal to the internal resistance of the voltage source before the Thevenin resistance is evaluated. If $R_{\text{internal}} = 0$, then replace the voltage source with a zero ohm resistor (short). This resistance is $R_{TH}$.

![Figure 3](image)

Let us consider the example shown in Fig. 3. After the sources are removed we can find the resistance “looking back” from the open terminals of the network by measuring the resistance with an ohmmeter connected to terminals $a$ and $b$. This is just like an equivalent resistance as we saw in the Kirchhoff lab. We can also calculate this resistance. It is easiest to calculate the equivalent resistance starting from the left side of the network shown in Fig. 3. We can see that $R_3$ is in parallel with $R_4$. Remember that the resistance of 2 resistors in parallel is:
\[ R_p = \frac{R_3R_4}{R_3 + R_4} \]  

This parallel resistance is in series with \( R_2 \). This gives us a Thevenin resistance of:

\[ R_{TH} = \frac{R_3R_4}{R_3 + R_4} + R_2 \]

Now we have the components we need to create the Thevenin equivalent circuit shown in Fig. 1. Next the load resistor is replaced and we can write the equations for the current and voltage this resistor is exposed to. Fig. 4 shows the Thevenin equivalent circuit with the load resistor, \( R \), replaced.

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\begin{align*}
\text{Step 3} \\
\text{From the Kirchhoff lab we know the current through } R \text{ is:} \\
I_R &= \frac{V_{TH}}{R_{TH} + R} \\
\text{and the voltage across } R \text{ is:} \\
V_R &= IR = \frac{V_{TH}R}{R_{TH} + R}
\end{align*}
\]

\( V_{TH} \) is the Thevenin equivalent voltage obtained in Step 1, \( R_{TH} \) is the Thevenin equivalent resistance obtained in Step 2, and \( R \) is the load resistor removed in Step 1.